Variability in earthquake stress drop and apparent stress

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[1] We apply empirical Green's function coda-based analysis to four earthquake sequences in Japan that span a magnitude range of 1.8 to 6.9, to measure radiated energy, corner frequency and stress drop. We find no systematic dependence of apparent stress or stress drop on seismic moment for these sequences, and find they both are log-normally distributed; however, we identify several anomalous events - both energetic and enervated - that show sharply different spectral signatures from the rest of the population. These events indicate that much of the variation in apparent stress and stress drop is statistically significant, which may have important implications for seismic hazard analysis. **Citation:** Baltay, A., S. Ide, G. Prieto, and G. Beroza (2011), Variability in earthquake stress drop and apparent stress, *Geophys. Res. Lett.*, *38*, L06303, doi:10.1029/2011GL046698.

1. Introduction

[2] The relationship between apparent stress and earthquake moment remains a controversial topic due to the difficulty in correcting for propagation effects over the broad frequency range required to measure the radiated energy [*Ide et al.*, 2003]. Some studies find no dependence of apparent stress on moment, while others observe a systematic increase in apparent stress with moment [e.g., *Walter et al.*, 2006]. Studies of stress drop and apparent stress indicate that the two vary together [e.g., *Abercrombie*, 1995; *Ide et al.*, 2003]. Earthquakes with higher stress drops will have more intense ground motions. Thus, if apparent stress scales with seismic moment, attenuation relationships might underestimate strong ground motion in large earthquakes.

[3] Large earthquake populations reveal strong variations in stress drop, but little in the way of systematic behavior or dependence on seismic moment [e.g., *Aki*, 1972; *Hanks*, 1977; *Allmann and Shearer*, 2009]. Because static stress drop measurements depend on the corner frequency cubed, small uncertainties in corner frequency map into large uncertainties in the stress drop, and it's often unclear how much of this variability is due to measurement error, rather than variability in source properties [*Sonley and Abercrombie*, 2006; *Prieto et al.*, 2007].

[4] We expand on *Baltay et al.* [2010] to measure seismic energy, corner frequency, and stress drop, using an empirical Green's function (eGf) coda based measurement that provides stable and robust source spectra. We apply this approach to four earthquake sequences in Honshu, Japan that are well recorded by seismic networks. As in *Baltay et al.* [2010], we find no systematic variation of apparent stress or stress drop with seismic moment; however, we do find several anomalous events with unusually high or low stress drops. The energetic events could have high stress drop and/or high rupture velocity, suggesting there could be a population of "rogue" earthquakes that have particularly intense strong ground motion for their size.

2. Empirical Green's Function Coda Spectrum Method

[5] We follow *Baltay et al.* [2010] to create coda-based source spectra with path effects removed through an eGf correction. We apply it to each station individually, then stack spectra over all stations for each earthquake.

[6] Narrowband envelopes are created from horizontally averaged displacement records. Non-dimensional coda spectra are constructed from the average envelope over a constant 20-second time window in the coda starting at the S-wave arrival. In Japan, we found a simple 20-second window reduced the inter-station scatter of scaled energy as much as the variable window lengths used by *Baltay et al.* [2010], due to both better station coverage and borehole recordings for the smaller events.

[7] We use the smallest event as an eGf, assuming an ideal Brune ω^{-2} spectrum, as in *Baltay et al.* [2010] Equation (1). For the eGf event we estimate corner frequency based on the Japan Meteorological Agency (JMA) magnitude and equation (3) of *Baltay et al.* [2010], assuming a stress drop of 3 MPa. These assumptions are made for the eGf event only. If we use *Boatwright*'s [1980] spectra model instead, apparent stress increases by about 10% for all events.

[8] The spectra for the other earthquakes are sequentially corrected to remove path effects. To convert the coda spectra to absolute displacement spectra, we use the independently determined seismic moment of each main shock, from the National Research Institute for Earth Science and Disaster Prevention (NIED) in Section 4, below. The corrected spectral level then sets the moment, and M_w, of each smaller event.

[9] We extrapolate the source spectra to the high and low frequencies and estimate the radiated seismic energy as

$$E_s = \frac{I}{4\pi^2 \rho \beta^5} \int_0^\infty \left| \omega \cdot \dot{M}(\omega) \right|^2 d\omega \qquad (1)$$

with $\dot{M}(\omega)$ the displacement source spectra, ρ = 2800 kg/m^3, β = 3600 m/s (except for the Kamaishi sequence, where β = 4400 m/s), and I, the average mean-squared S-wave radia-

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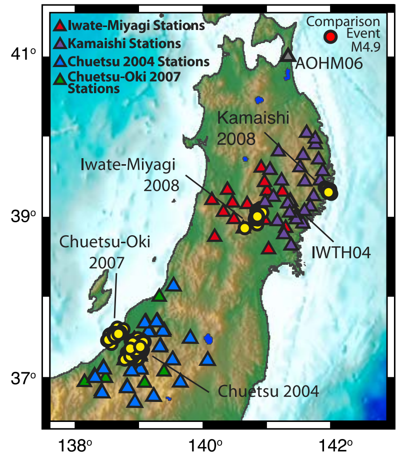

Figure 1. Event (yellow circles) and station (triangles) locations. Stations from two studies may overlap. Kamaishi comparison event (Figure 4) in top right (circle).

tion pattern coefficient, assumed to be 2/5 [*Boore and Boatwright*, 1984].

3. Source Parameter Estimation

[10] Once we determine the stacked source spectra, we model the spectral shape to determine the corner frequency. Each spectrum is fit with a Brune ω^{-2} model

$$u(f) = \frac{M_o}{1 + (f/f_c)^2}. \quad (2)$$

The corner frequency is found by minimizing the L-2 norm of the residuals. Using the circular crack model of *Eshelby* [1957] and the relationship between source dimension and corner frequency from *Brune* [1970], stress drop, moment and corner frequency are related as

$$\Delta\sigma = \frac{7M_o}{16}\left(\frac{2\pi f_c}{2.34\beta}\right)^3. \quad (3)$$

[11] We compare Brune static stress drop ($\Delta\sigma$) to apparent stress, ($\tau_a = \mu\, E_s/M_o$) [*Wyss and Brune*, 1968]. Using the above assumptions under self-similarity, the theoretical relationship is

$$\frac{\Delta\sigma}{\tau_a} = 4.3 \quad (4)$$

[*Singh and Ordaz*, 1994].

[12] For the larger events, we fit a Brune spectral model to measure the corner frequency. Fitting with a Boatwright spectral model yields negligible differences; however, the relationship between corner frequency and stress drop is model dependent [e.g., *Sonley and Abercrombie*, 2006]. Here we use a Brune stress drop, which allows for a straightforward comparison of our results with other studies.

4. Four Study Areas in Honshu, Japan

[13] Our eGf method requires earthquakes close enough to share common path effects. We analyze four sequences in northern Honshu, Japan including three large main shock-aftershock sequences (Figure 1). We use only co-located, borehole recordings from Hi-net broadband and KiK-net strong motion network, which allows us to extend the analysis to higher frequencies and smaller magnitudes than possible from surface observations. In each sequence, only events recorded at 10 or more stations are analyzed, with 17 to 18 stations total for each sequence, an aggregate of 89 events (see Table S1 for event list).[1]

[14] The 2004 M_w 6.6 Chuetsu (mid Niigata) earthquake occurred in a highly active area of western Honshu at a depth of 13 km. The aftershock sequence was especially rich, with events of M_{JMA} 6.5, 6.3 and 6.0, and hundreds of smaller and intermediate-sized events, occurring on a complex system consisting of five reverse faults [*Hikima and Koketsu*, 2005]. We analyze 32 events total, at 18 stations. The mainshock moment determined by NIED is 7.5×10^{18} N-m, or M_w 6.55.

[15] The 2007 M_w 6.7 Chuetsu-Oki sequence received much attention due to its impact on the Kashiwazaki Kariwa nuclear power plant. The main shock and aftershocks occurred offshore Niigata prefecture on a shallow, southeast dipping thrust fault, with depths of 15–20 km [*Miyake et al.*, 2010]. We analyze 15 events in this sequence, recorded at 17 stations, with a mainshock seismic moment of 1.42×10^{19} N-m, or M_w 6.73 (NIED).

[16] The 2008 M_w 6.9 Iwate-Miyagi Nairiku earthquake occurred between the Iwate and Miyagi prefectures in central northern Honshu at 8 km depth on a shallow inland crustal reverse fault, and is well known for a recorded acceleration in excess of 4 g [*Yamada et al.*, 2009]. The moment of the main shock is 2.7×10^{19} N-m (NIED), M_w 6.92. We analyze 27 aftershocks at depths up to 12 km, recorded at 17 stations.

[17] The repeating earthquake sequence offshore of Kamaishi, Iwate, has a main shock magnitude of 4.9 +/− 0.2 and recurs every 5 to 6 years. Between repeats, many smaller earthquakes rupture similar patches located on the deepest part of the inter-plate main thrust zone, at about 50 km depth [*Uchida et al.*, 2010]. The 2008 Kamaishi main event (08:00 on 01/11/2008) has moment 1.035×10^{16} N-m, M_w 4.64 [*Coordinating Committee for Earthquake Prediction in Japan*, 2008]. We use the 16 events that have occurred since the local installation of 17 Hi-net stations in the area in 2002.

5. Radiated Energy and Apparent Stress

[18] Scaled energies, E_S/M_o, for each sequence are compared to previous studies (Figure 2 and Table S1). In all four sequences, the apparent stress shows no significant trend with moment. The best-fit parameters include zero-slope (no scaling) within the uncertainties. In particular, the radiated energy of the Chuetsu main shock is 2.9×10^{14} J, which falls between the USGS estimate of 1.4×10^{14} J (following *Boatwright and Choy* [1986]) and the estimate of 3.2×10^{14} J from *Izutani* [2005]. Our energy results and independent moment calculations from two of the larger aftershocks (*M*5.7 and *M*5.3) compare closely with those of *Izutani* [2005]. Energy of the Chuetsu Oki 2007 main shock is 1.9×10^{15} J, larger than the USGS estimate of 1.4×10^{14} J, as is that of the Iwate-Miyagi main shock with

[1] Auxiliary materials are available in the HTML. doi:10.1029/2011GL046698.

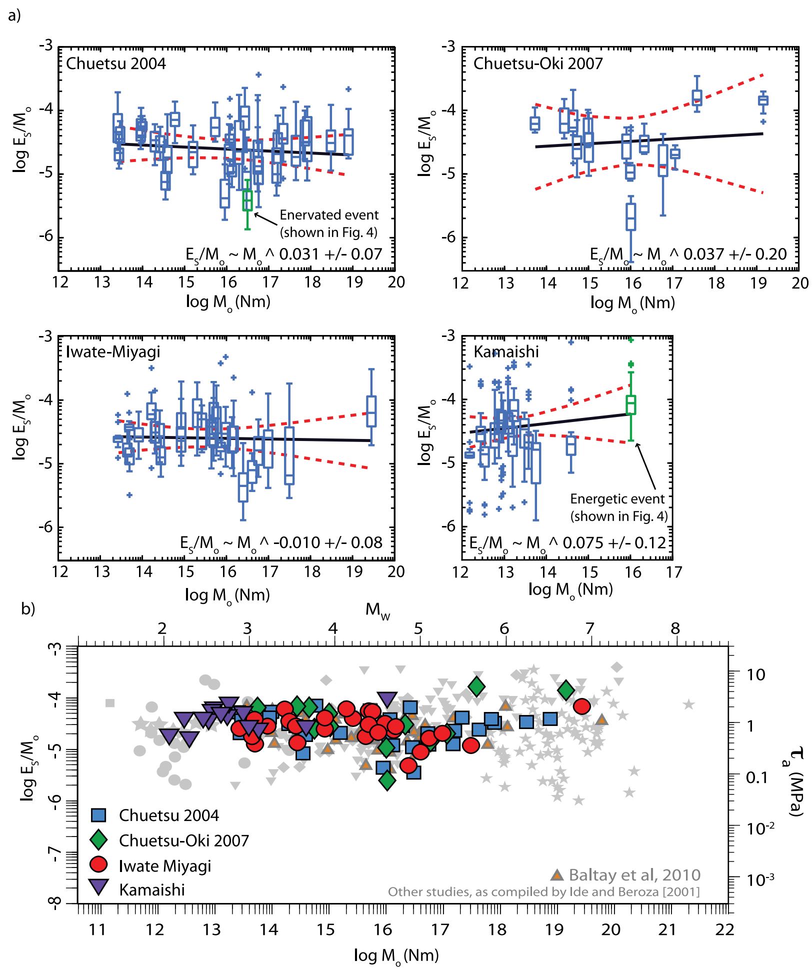

Figure 2. (a) Inter-station data range given by the non-parametric box and whisker plot: middle bar is the median; box indicates the 25th and 75th percentiles, the difference of which is the interquartile range; whiskers show distance to the farthest data point within 1.5× interquartile range; and pluses are outlying data. For most events, whiskers are small with few outliers, showing tight inter-station scatter. Black line is L2 norm fit of the mean scaled energy; red dashed lines are 95% confidence on the fit parameters. (b) Scaled energy and apparent stress for all events taken together, overlain on *Ide and Beroza* [2001] and *Baltay et al.* [2010] from the western United States.

a radiated energy of 1.8×10^{15} J compared to USGS energy estimate of 2.6×10^{14} J. In some of the sequences, we find that the main shock has higher energy than most aftershocks.

[19] Figure 2b reveals no obvious trend of increasing apparent stress over eight orders of magnitude in seismic moment. These four sequences follow the same scatter and mean as those of *Baltay et al.* [2010] for sequences in the Western US as well as the compilation of *Ide and Beroza* [2001]. Overall, apparent stress has a log mean of 1.15 MPa and log standard deviation of 0.95, very consistent with many previous studies that find an apparent stress near 1 MPa [*Ide and Beroza*, 2001].

6. Stress Drop

[20] We find that Brune stress drop is log-normally distributed with a mean of 5.92 MPa. There is no dependence of stress drop on moment, however there are several events with stress drops higher than average, as well as events with very low stress drops (Figure 3 and Table S1). The Kamaishi sequence has higher stress drops, which may be due the deeper location of the events. While the mean value of stress drop is dependent and positively correlated with the assumed 3 MPa stress drop of the eGf, the distribution, relative variations, and the lack of scaling are not affected by the assumed eGf parameters.

[21] Our comparisons of stress drop and apparent stress are generally consistent with the relationship expected for constant stress, with the exception of two enervated events. The departure of some events from the expected ratio $\Delta\sigma/\tau_a = 4.3$ is due to higher corner frequencies of the spectra, which render higher stress drops than expected given the apparent stress. When considering the error, almost all of the events fall on the expected line (Figure 3).

[22] The Kamaishi stress drop results are consistent with other findings on this repeating sequence. *Uchida et al.* [2010] found the smaller earthquakes in the sequence to have stress drops between 3 and 11 MPa, with a stress drop of 27 MPa for the 2008 main event. We find a stress drop of 28.72 MPa for the same 2008 main event, and a mean of 9.59 MPa for the rest of the Kamaishi events.

[23] Our findings of non-scaling stress drop with moment, but with variations in stress drop values, are consistent with

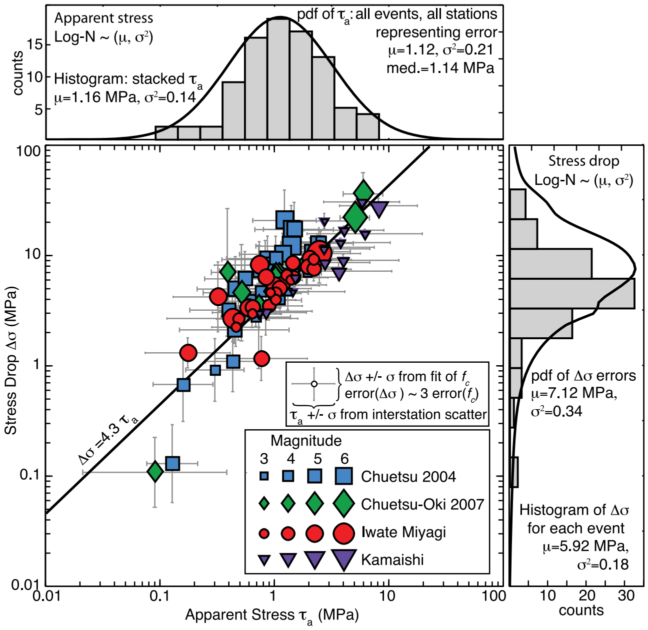

Figure 3. Apparent stress compared to stress drop with error bars, showing theoretical relation in black line. Enervated earthquakes have lower stress drop and apparent stress than expected (lower left) while energetic earthquakes have higher stresses (upper right). Histograms show log-normal distributions of events. Solid lines include the effect of station-to-station variation (apparent stress) and uncertainties in measurements (stress drop).

other studies. For global earthquakes from M_w 5.2 to 8.3, *Allmann and Shearer* [2009] estimated stress drops ranging from 0.3 to 50 MPa, with the median value of 4 MPa independent of moment. *Oth et al.* [2010] find a mean Brune stress drop of 1.1 MPa for crustal events ranging from M_{JMA} 2.7–8 in Japan, with stress drops ranging from 0.1 to nearly 100 MPa.

7. Energetic and Enervated Events

[24] While our results point to a lack of scaling of either apparent stress or stress drop with moment, we find statistically significant departures from the average. Overall, both apparent stress and stress drop are log-normally distributed, as are their errors. This is not due to any shortcomings in the measurements, but rather because earthquakes do display real variations. To emphasize this variation, we highlight two particular events: an energetic event, as well as an enervated earthquake.

[25] The M_w 4.64 Kamaishi main event from 2008 (Figure 4, left) is highly energetic. The broadband record is similar in both amplitude and signature to a typical event at a similar depth on the same plate interface (Figure 4, middle left); however, the low frequencies of the Kamaishi event are depleted, while the high frequencies of the energetic Kamaishi earthquake are enriched. The main Kamaishi event has an apparent stress of 8.2 MPa, the highest of any event in this study, and a stress drop of 28.7 MPa.

[26] The M_w 4.95 enervated event from the Chuetsu 2004 sequence (Figure 4, middle right) has an apparent stress of only 0.12 MPa and low stress drop of 0.12 MPa, due to a low corner frequency of 0.24 Hz. While the low frequency record of the enervated event is very similar, in both amplitude and phase, to a standard event (Figure 4, right), the broadband record has a different signature and a slower start than a typical event. The high frequencies of the enervated event are completely depleted. A second enervated event from Chuetsu 2007 of *Mw* 4.64 has an apparent stress of 0.09 MPa and stress drop of 0.11 MPa, with a similarly low corner frequency of 0.31 Hz. These low values are comparable to those found for slow oceanic transform fault earthquakes [*Perez-Campos*, 2003].

[27] These enervated and energetic events are not outliers due to difficulties or shortcomings in data processing; rather, their waveforms confirm that they are genuinely anomalies with energies and stress drops well outside the main population. While the enervated events may be scientifically interesting slow earthquakes, they are of little concern for seismic risk. The energetic events, however, have more high-frequency energy than otherwise predicted for their size. Understanding the origin of this high-frequency energetic event is important for accurate prediction of strong ground motion at low probability thresholds. The possibility that main shocks may also be slightly more energetic than their aftershocks could be important in hazard assessment.

8. Conclusion

[28] We estimate radiated energy and apparent stress of four earthquake sequences in Honshu, Japan, ranging from magnitude 1.8 to M_w 6.9, using the methodology of *Baltay et al.* [2010]. The spectra are modeled to measure corner frequency and Brune stress drop. Overall, we find no dependence of apparent stress or stress drop on moment. The confidence on the best best-fit linear relationship between $\log_{10} E_S$ and $\log_{10}$ apparent stress includes a zero-slope, no dependence on moment, for each sequence. Apparent stress and stress drop follow the expected theoretical relationship. For all four sequences in aggregate, the mean apparent stress is 1.15 MPa, the scaled energy ratio is 2.96×10^{-5}, and the mean stress drop is 5.92 MPa.

[29] Our results support a self-similar earthquake model and current practice of using a constant stress drop assumption for strong ground motion prediction. By utilizing the coda waves and averaging over many stations, we make robust estimations of the energy and stress drop of each event. Observations of scaled energy with moment show no significant statistical departure from zero dependence. Many other studies that find a dependence of scaled energy on moment use small data sets or do not offer a statistical analysis sufficient to gauge the significance of their results. Furthermore, selection bias can introduce artificial trends in stress drop or energy with moment [*Ide and Beroza*, 2001; *Oth et al.*, 2010].

[30] We show that both apparent stress and stress drop are log-normally distributed, and identify several enervated and energetic earthquakes. These anomalous earthquakes represent statistically significant differences in energy, apparent stress and stress drop. While the majority of our events follow a constant apparent stress, the anomalous events are outside of these ranges and are not predictable simply given their moment. Further understanding of these events will be important to completely quantify their associated hazard.

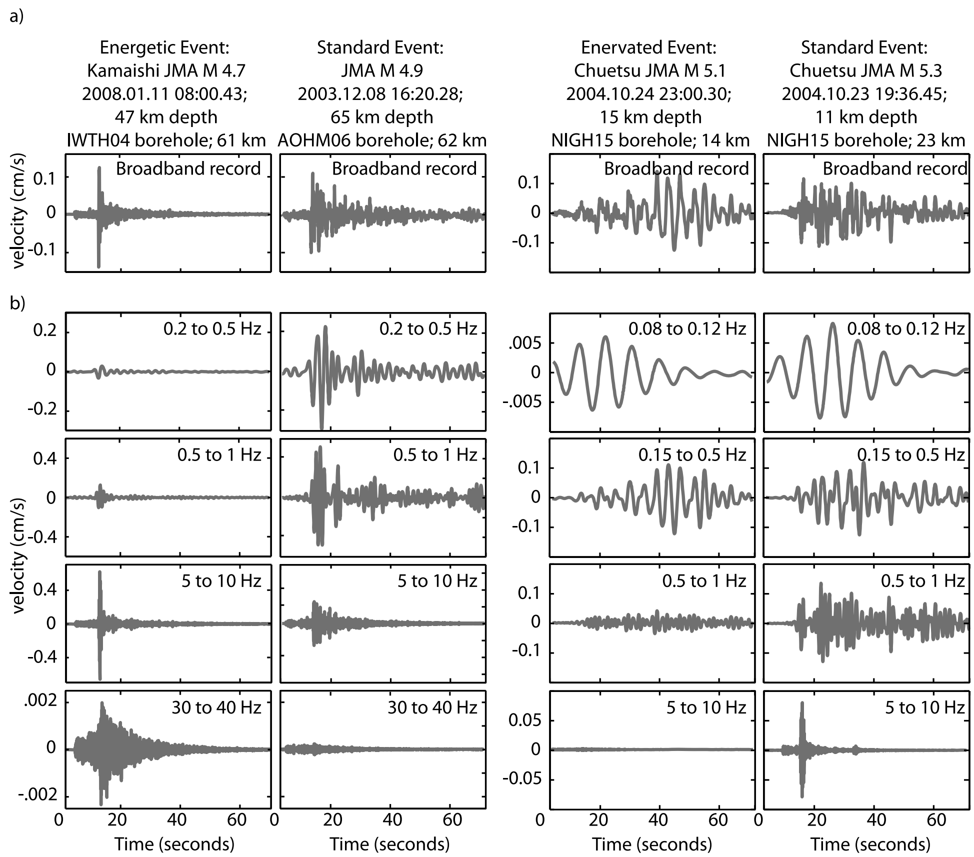

Figure 4. Anomalous energetic and enervated events: (left) Kamaishi 2008 main event and (middle right) enervated event from Chuetsu 2004 sequence. (middle left and right) Comparison events with same mechanism, similar magnitude, and similar event-station distance. (a) Broadband borehole velocity seismograms, and (b) bandpassed seismograms. Pass bands are different in the energetic vs. enervated series to highlight the different trends.

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