MULTITAPER WIGNER-VILLE SPECTRUM FOR DETECTING DISPERSIVE SIGNALS FROM EARTHQUAKE RECORDS

Germán A. Prieto, Frank L. Vernon, Guy Masters, David J. Thomson

Institute of Geophysics and Planetary Physics
University of California, San Diego, La Jolla, CA 92093
Queen's University, Kingston Ontario,
gprieto@ucsd.edu (858) 534-8766

ABSTRACT

In earthquake record analysis we deal with transient non-stationary records corresponding to the seismic waves generated during the rupture process. Dispersive surface waves recorded at seismic stations contain information about the propagation medium, and thus are used to look at the structure of the Earth. Surface wave observations are commonly used to study crustal and upper-mantle structure of the Earth [1]. The resolution of the models obtained depend on the accuracy of determining group or phase velocity as a function of frequency. A desirable outcome of such an analysis is to obtain not only the fundamental mode surface wave, but also to obtain the higher mode surface wave dispersion, to help constrain crustal structure. We use a multitaper version of the Wigner-Ville spectrum to increase resolution of the dispersive signals at all frequencies. We use both synthetic and actual seismic signals to test the technique.

1. INTRODUCTION

Dispersive surface wave observations are commonly used to study upper-mantle and crustal structure of the Earth. There are different approaches to surface wave tomography, and most often obtained through the calculation of spectrograms, a moving window method [2], or through multiple filter analysis MFA [3]. All these methods have the basic shortcoming of smearing in the time-frequency plane, either losing resolution in the frequency domain (spectrograms) or in the time domain (MFA). We test different time-frequency methods to improve readability of dispersive signals, allowing for correlations between distant frequencies, thus concentrating linear features in the time-frequency plane.

We show that the dual-frequency spectrum [4, 5] is a useful and practical tool and can be used to obtain a time-frequency description of the signal, with the advantage of obtaining an estimate with stronger resistance to spectral leakage than other standard time-frequency methods. As an example, synthetic and actual Rayleigh waves are used to show dispersion curves.

2. SPECTRAL PROPERTIES

Assume we are given N contiguous samples of a discrete time process $x(t)$ for $t = 0, 1, \ldots, N-1$. Assume that the process is harmonizable [6], so that it has a Cramér Spectral representation

$$x(t) = \int_{-1/2}^{1/2} e^{2\pi i f t} dZ(f) \tag{1}$$

where the measure $dZ(f)$ is the complex valued increment field, or generalized Fourier transform, or orthogonal incremental process of the process $x(t)$.

Consider the covariance function (assume $x(t)$ is a zero-mean process)

$$\Gamma(t_1, t_2) = E[x(t_1)x^*(t_2)] \tag{2}$$

where superscript $*$ indicates complex conjugate. The covariance then is

$$\Gamma(t_1, t_2) = \iint e^{2\pi i(t_1 f_1 - t_2 f_2)} \gamma(f_1, f_2) df_1 df_2 \tag{3}$$

the two-dimensional Fourier transform of a generalized spectral density, defined by

$$\gamma(f_1, f_2) df_1 df_2 = E[dZ(f_1) dZ^*(f_2)] \tag{4}$$

If the incremental process $dZ(f)$ are orthogonal, that is no correlation at distant frequencies, we would have

$$\Gamma(t_2 - t_1) = \int e^{2\pi i(t_2 - t_1) f} S(f) df \tag{5}$$

and we obtain the well known stationary case result, that the covariance function is only a function of $(t_2 - t_1)$.

But, if the signal is non-stationary, equation (4) would describe the essential feature of non-stationary processes, namely that there is correlation between signal elements at different frequencies. The generalized spectral density $\gamma(f_1 f_2)$ is also known as the dual-frequency spectrum (DFS) or sometimes called Loève spectrum. Correlation among different frequency components is responsible for non-stationarity.

The *Wigner-Ville Spectrum* (WVS) [7] admits two equivalent formulations

$$W(t,f) = \begin{cases} \int_{-\infty}^{\infty} \Gamma(t + 1/2\tau, t - 1/2\tau) e^{-2\pi i f \tau} d\tau \\ \int_{-\infty}^{\infty} \gamma(f + 1/2g, f - 1/2g) e^{+2\pi i g t} dg \end{cases} \tag{6}$$

One can then in practice obtain the WVS from the covariance function or the DFS. When dealing with colored signals, with large dynamic ranges over a short frequency range, estimates for discrete signals suffer from spectral leakage. It is known that the periodogram approach, based on the covariance function, has severe spectral leakage problems. If the DFS can be estimated with good bias reduction properties, one would expect the associated WVS to also provide good protection against spectral leakage, compared to the time domain derived spectrum.

3. ESTIMATION OF DUAL-FREQUENCY AND WIGNER-VILLE SPECTRUM

Assume we have a sampled random process $x(t)$ at temporal instants $t = 0, 1, \ldots, N-1$. We assume then that the process is harmonizable and has the spectral representation (1). We use the multitaper approach [8] to get a stable estimate of the DFS. The eigencoefficients of the multitaper expansion for a given choice of time and bandwidth N, W are

$$y_k = \sum_{t=0}^{N-1} x(t) v_k(t) e^{-2\pi i f t} \tag{7}$$

for $k = 0, 1, \ldots, (2NW-1)$, where $v_k(t) = v_k(N, W; t)$ is the kth Slepian sequence and is implicitly a function of the choice of N and W. We use an adaptive weighted approach in order to estimate the spectrum

$$S(f) = \frac{\sum_{k=0}^{K-1} \lambda_k d_k^2(f) |y_k(f)|^2}{\sum_{k=0}^{K-1} d_k^2(f)} \tag{8}$$

where d_k are the weights of each spectral estimate, λ_k are the eigenvalues associated with each taper and y_k the discrete Fourier transforms of the tapered data (7). The weights are determined using an iterative process for the time series.

The DFS is then estimated by

$$\gamma(f_1, f_2) = \frac{\sum_{k=0}^{K-1} \lambda_k d_k(f_1) y_k^*(f_1) d_k(f_2) y_k(f_2)}{\left[\sum_{k=0}^{K-1} d_k(f_1)^2\right]^{1/2} \left[\sum_{k=0}^{K-1} d_k(f_2)^2\right]^{1/2}} \tag{9}$$

and it can also be shown that an estimate of the dual-frequency coherence

$$\gamma^2(f_1, f_2) = \frac{|\gamma(f_1, f_2)|^2}{S(f_1) S(f_2)} \tag{10}$$

has a range between 0 and 1 [5, 8, 9]. It is straightforward to extend this estimate to cross-spectrum and cross-coherence between two distinct signals.

Once the DFS is estimated one can estimate the Wigner-Ville spectrum by a simple axis rotation followed by a 1-D Fourier transform of the DFS taken perpendicular to the $f_1 = f_2$ diagonal. The multitaper approach stabilizes and effectively reduces variance by means of averaging estimates that are statistically independent due to the orthogonality of the discrete prolate spheroidal sequences.

For dispersive signals, much of the useful information lies in the frequency combinations very near the main diagonal, the interference terms of closely located frequency bins. To reduce interference between distant frequencies we applied a Gaussian filter to the rotated DFS before taking the Fourier transform. The width of the filter was chosen after experimenting with different values in order to taper the correlations between distant frequencies . The result is an enhancement of the dispersive signals, reduction of interference terms between, for example, distant line components, and with the trade-off of losing some time resolution in the resultant WVS.

4. SEISMIC SIGNALS

We use both synthetic and actual seismic signals to test whether the Wigner-Ville spectrum (or any of the family of time-frequency distributions associated with the Wigner-Ville Distribution) has the potential to improve estimates of crustal and mantle structure by detecting higher mode surface wave dispersion. One of the main problems is that the amplitude of the fundamental mode is in many cases larger than that of the higher modes, obscuring the dispersion of the higher modes and introducing high amplitude interference terms. The spectrum of a typical recorded teleseismic signal (Figure 1) has a large dynamic range.

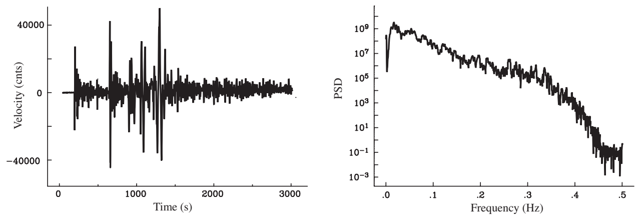

Fig. 1. Record from the September 26 2005 Peru M7.5 Earthquake recorded at station PFO and multitaper power spectrum estimate. Note the high dynamic range of the signal. Signal has 3060 data points and we chose time-bandwidth product $NW = 4$.

As a simple test, we generate synthetic seismograms from an anisotropic Earth model [10] with expected time-frequency dispersion curves shown in the lower panel of Figure 2. Note that the higher overtone curves have similar arrival times and are thus difficult to separate and measure directly. The interference terms between the fundamental mode and the higher modes are quite simple, but other interferences are not easy to isolate.

Figure 3 shows results obtained by using the multitaper approach on data from the ANZA seismic network [11] in Southern California, specifically the vertical components from station PFO, The specific event is a magnitude 7.0 earthquake located near the Southern Mariana Islands in October 12 2001. The data was sampled at 1 Hz. A unfiltered and filtered version of the WVS are shown.

A final example is shown in Figure 4. The recent September 26 2005 Peru earthquake, 116 km depth, similar to the synthetic model created, shows both the fundamental mode and possibly the first of the higher modes. More of the interference terms are present, due to noise in the signal, especially cross terms associated with the fundamental mode.

5. CONCLUSION

A multitaper approach is proposed to estimate the dual-frequency spectrum and the Wigner-Ville spectrum, both with good bias reduction properties. A frequency domain filter was applied to obtain a reduction in the interference terms for the WVS. The proposed method was applied to both synthetic and real seismic signals to investigate the potential of the WVS to be used in estimating group and phase velocity of dispersive surface waves. The WVS is capable of describing the fundamental mode over a very wide frequency range, as well as showing contributions from the higher modes. In the frequency range where most of the energy is concentrated (20 - 60 mHz) the higher modes have similar arrival times, making it difficult to isolate them individualy. Other time-frequency distributions such as the Rihackzek Distribution or the reassigned spectrogram which can be obtained with the multitaper approach could potentially yield better estimates of the dispersion.

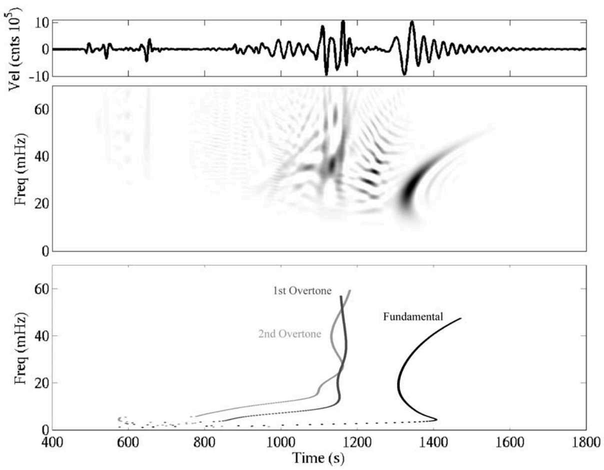

Fig. 2. Wigner-Ville spectrum of a synthetic seismogram created using the anisotropic Reference Earth model [10] with significant signal up to 200 mHz from a 136 km deep teleseismic earthquake about 5000 km distance from source to receiver. The top panel shows the synthetic signal used in this test, middle panel shows the multitaper WVS and the lower panel the expected time frequency behavior of the dispersive waves.The fundamental mode is quite obvious and the first two overtones are also visible in the figure. Interference terms complicate the analysis of the dispersive curves and a separation between the first and second overtone is difficult.

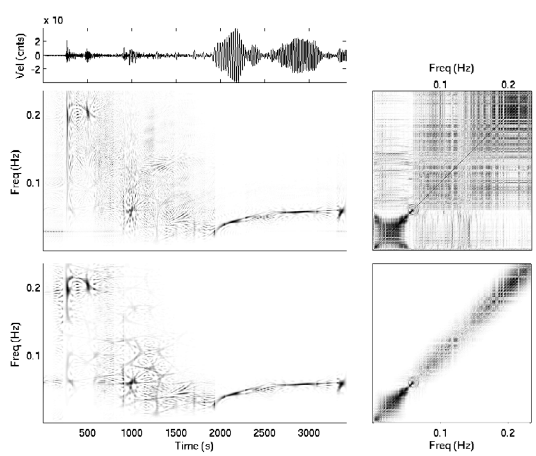

Fig. 3. WVS estimation of the trace recorded at a vertical component from station PFO in California for a Mariana Island earthquake in 2001. The top panel shows the seismic signal as a function of time, the two plots below show the WV and filtered WV spectra and associated dual-frequency coherences on the right. Many of the interference terms have been reduced by the filtering approach.

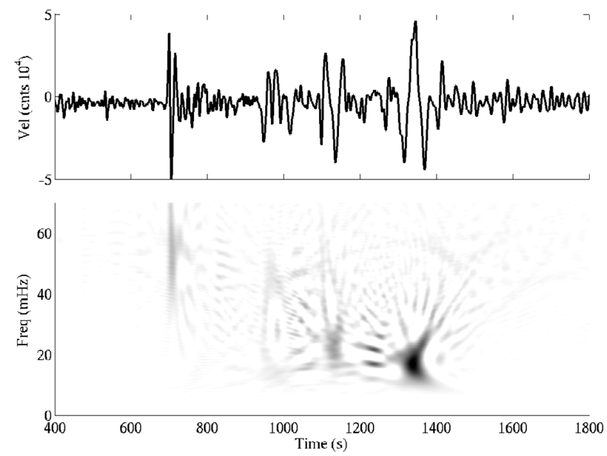

Fig. 4. Wigner-Ville spectrum of the September 26 2005 Peru M7.5 Earthquake. The depth of this earthquake is similar to that of the synthetic model used. The fundamental and the first overtone are present in the WVS, at approximately the theoretical locations shown in lower panel of Figure 3.

6. REFERENCES

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